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# FLOW RATING ANALYSIS PROCEDURES FOR PUMPS

**Muluneh Imru**
**Youchao Wang**

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**South Florida Water Management District**
**3301 Gun Club Road**
**West Palm Beach, FL 33406**

## EXECUTIVE SUMMARY

This report describes the procedures of flow rating analysis for pumps adopted by the South Florida Water Management District (District). Analysis of available measurements, determination of additional measurements required, existing flow estimation method evaluation, development and calibration of a new rating equation are presented with pump stations S140_P and S9_P as examples.

Water flows through pumps in either of two ways, pumping or siphoning, depending on headwater and tailwater elevations. Different equations are used to estimate flow through pumping and siphoning. For flow computation purposes the District pumps are classified into eight categories (cases). Initially, the U. S. Army Corps of Engineers (USACE) developed and calibrated flow models for the first six cases. Subsequently, two pump flow cases were developed and calibrated by engineers in the District.

When enough discharge data points have been obtained, that is, 15 or more points distributed over a wide range of pump operation settings, the measured discharges are compared against the data computed by the District's discharge computation program (FLOW). If the comparison results show average absolute relative errors of more than 10%, the existing rating equation is considered to be not satisfactory and modifications to the existing equation or development of a new one is essential for better flow estimation accuracy.

New rating equations are developed based on principles of conservation of mass and energy and pump affinity laws. Calibration of new rating equations involves regression analysis using the Least Square Method. Regression coefficients of a new rating for the example (S9_P) were determined using available measurements and pump performance curves.

The report provides the summary and recommendations of flow rating analysis procedures for pumps. The summary and recommendations are brief outlines of the steps for rating analysis and impact evaluation for pump stations.

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## LIST OF ACRONYMS AND ABBREVIATIONS

| | |
|---|---|
| CERP | Comprehensive Everglades Restoration Plan |
| DBHYDRO | Hydrometeorologic and Water Quality Database |
| District | South Florida Water Management District |
| DCVP | Data Collection/Validation Preprocessing |
| IWEB | District's internal Internet Website |
| Qmeas | Database table containing measured flow data |
| Qmr | Program that ranks errors at a station per range of operation |
| Qverify | Program that compares measured and computed discharge records |
| SQL | Structured Query Language |
| USACE | U. S. Army Corps of Engineers |

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## 1. Introduction

The South Florida Water Management District (District) maintains and operates over 400 hydraulic structures, including spillways, pump stations, culverts, and weirs (Ansar et al, 2003). With the Comprehensive Everglades Restoration Plan (CERP) and other regulatory programs, the number of structures requiring flow monitoring is expected to increase over the next ten years.

Water flows through pumps in either of two ways, pumping or siphoning, depending on headwater and tailwater elevations. Different equations are used to estimate flow through pumping and siphoning. For flow computation purposes, District pumps are classified into eight categories (cases). Initially, the U. S. Army Corps of Engineers (USACE) developed and calibrated flow models for the first six cases. Subsequently, two pump flow cases were developed and calibrated by engineers in the District.

The input data used by FLOW (a program developed in-house to compute discharges at control structures) for a pump station are instantaneous values of upstream and downstream stages and engine speed, which are stored in the District's Data Collection/Validation Preprocessing (DCVP) database. Discharge data obtained through streamgauging are entered into streamgauging database (Qmeas) tables. When enough discharge data points have been obtained, that is, 15 or more points distributed over a wide range of pump operation, the measured discharges are compared against the data computed by FLOW (Akpoji et al, 2003). Based on the existing stream flow measurements, the relative errors in discharge can be obtained using Qverify (a program that compares measured and computed discharge records per station).

The main goal of rating analysis is to provide the best quality flow data for each pump. When running Qverify, if the results show an average absolute relative error more than 10%, the existing rating is considered to be unsatisfactory and modifications to the existing rating equation or development of a new one is initiated. Discharge data verification and rating calibration are performed repeatedly until an optimum correlation is obtained between measured and computed flows. The modification to the existing rating equation or development of a new one is based on physical principles such as the conservation of energy, continuity, and pump affinity laws (Appendix A).

Major steps of a typical pump flow rating analysis are presented in subsequent sections of this publication. Section 2 outlines the objective of the procedures for rating analysis at pump stations. Stream flow measurements and existing flow estimation procedures are described in Sections 3 and 4, respectively. Sections 5 and 6 discuss evaluation of the existing flow equation and determination of need for a new rating equation, respectively. Development of a new rating equation is discussed in Section 7 and calibration of the new rating equation is described in Section 8. Sections 9 and 10 provide conclusion and recommendations, respectively.

## 2. Objective and Scope

The objective of this report is to provide guidelines for evaluating existing rating equations and developing new rating equations that improve flow calculations and reduce relative errors of pump flow data. This report presents procedures for performing rating analysis and calibration at pump stations by using streamflow measurements and pump performance curves.

## 3. Stream Flow Measurements

Reliable streamflow measurements are essential to calibrate existing flow rating equations or to develop and calibrate new ones. A minimum of five data points which cover 2 ft of change in head differential or 25% of operating head differential range are required for initial calibration (Whalen et al., 1996). According to Hanna, 1999, it is recommended that data be gathered at not less than seven points, which should include the design rating and shutoff points.

### 3.1 Available Measurements

Discharge data, obtained through streamgauging, are entered into the streamgauging database (Qmeas) tables. The available measurements for pump stations can be obtained by running Structured Query Language (SQL) scripts (Appendix B). The following steps are necessary to analyze the available measurements.

- Separate the measurements into different groups based on pump type (constant-speed or variable-speed) and design flow capacity.
- Determine the measured discharge per unit based on the total number of pumps operating at the time of measurement.
- Calculate the head differential for each measurement based on the headwater and tailwater elevations.
- Plot the head differential against the engine speed. Figure 1 shows the distribution of the available measurements for pump station S140_P.

### 3.2 Additional Measurements Required

Based on the total number of available measurements for each pump station, we develop a streamgauging plan setting priorities and schedules for each pump station. Higher priority is given to those pump stations with few measurements. When attempting to set priorities for existing District pumps, for example, three groups were defined for all pump stations depending on the availability of data. The first group was for those with total measurements less than or equal to 5; the second group for those with measurements from 6 to 10; and the third group was for those with measurements more than 10.

The following steps will help to determine the additional measurements needed for rating analysis.

- Determine the design engine speed (available on the District's iweb/structure books).
- Find the maximum and minimum values for headwater elevation through DBHYDRO (hydrometeorologic and water quality database), and denote by $U_{max}$ and $U_{min}$, respectively.
- Find the maximum and minimum values for tailwater elevation through DBHYDRO, and denote by $D_{max}$ and $D_{min}$, respectively.
- Estimate the maximum and minimum head differential and denote by $\Delta H_{max}$ and $\Delta H_{min}$, respectively ($\Delta H_{max} = D_{max} - U_{min}$; $\Delta H_{min} = D_{min} - U_{max}$).

- Divide the head differential range into three different levels and denote low, medium, and high, respectively.
- Get the number of measurements required per range of operation for the selected pump station by running Qmr (a program that ranks errors at a station per range of operation).
- Determine the additional measurements required for selected pump station (Figure 1).

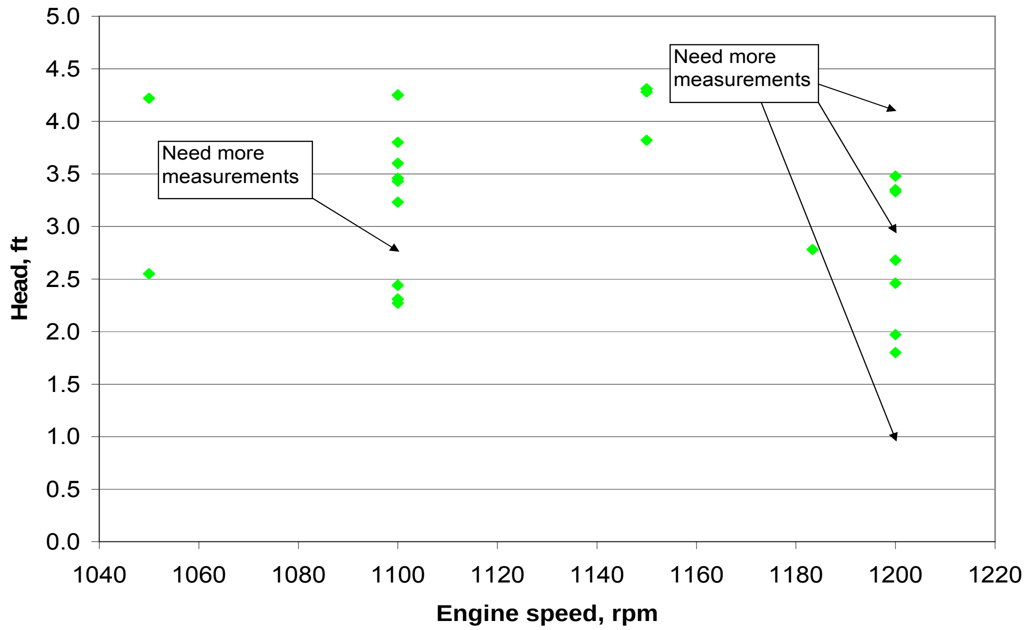


Figure 1. Relationship of head and engine speed for pump station S140_P

Table 1 is an example of the summary of the available and required additional measurements for the selected pump stations.

Table 1. Available and required additional measurements for the selected pump stations.

| Pump Station | Design Engine Speed (rpm) | Range of Head Differential (ft) | Measurements | | | |
|---|---|---|---|---|---|---|
| | | | RPM | Available | Required | Priority |
| S140_P | 1200 | 0.0≤DIFF≤2.0 | 1050≤RPM≤1200 | 2 | 3 | 3 |
| | | 2.0<DIFF≤4.0 | 1050≤RPM≤1200 | 16 | 1 | |
| | | 4.0<DIFF≤6.0 | 1050≤RPM≤1200 | 6 | 1 | |
| G310_P | Electric: 440 | 3.0≤DIFF≤6.7 | 440 | | 5 | 1 |
| | | 6.7<DIFF≤8.3 | 440 | 3 | 2 | |
| | | 8.3<DIFF≤10.0 | 440 | 1 | 4 | |
| G310_P | Diesel 470-cfs: 720 | 3.0≤DIFF≤6.7 | 720 | | 5 | 2 |
| | | 6.7<DIFF≤8.3 | 650≤RPM≤720 | 6 | 1 | |
| | | 8.3<DIFF≤10.0 | 720 | 1 | 4 | |
| G310_P | Diesel 950-cfs: 720 | 3.0≤DIFF≤6.7 | 720 | | 5 | 2 |
| | | 6.7<DIFF≤8.3 | 700≤RPM≤720 | 5 | 1 | |
| | | 8.3<DIFF≤10.0 | 720 | 3 | 2 | |
| S9_P | 733 | 1.0≤DIFF≤5.3 | 733 | | 5 | 3 |
| | | 5.3<DIFF≤9.7 | 650≤RPM≤733 | 28 | 1 | |
| | | 9.7<DIFF≤14.0 | 650≤RPM≤733 | 12 | 1 | |

Note: Priority numbers, 1, 2, 3 shown in Table 1 are based on the number of available measurements

## 4. Existing Flow Estimation Procedures

For flow computation purposes, District pumps and corresponding flow equations are classified into eight cases. The brief descriptions and equations for all eight cases are presented in Appendix C. Cases 1 and 5 are for constant-speed pumps, where flow is computed using a third order polynomial fitting of the head difference between upstream and downstream stages. Cases 2 and 4 are for highly-variable speed pumps, where flow is computed using a two-variable polynomial fitting of the pumping head and the engine speed of the pump. Case 3 is for variable-speed pumps, where flow is obtained from an interpolation between an upper and a lower discharge curves that are given by third order polynomials of the pumping head, as in Cases 1 and 5. The weighting coefficient of this interpolation is a function of the pump engine speed. Case 6 was developed for the variable-speed pumps at G600 and ACME1, where the flow is computed as a function of the pumping head and the pump engine speed. For Case 7, pump affinity laws are used to predict the discharge (Ansar et al, 2003). Case 8 is also a model based on pump affinity laws that can be used to estimate flow through variable speed pumps.

The following steps are useful to estimate the flow by using the existing rating equations.

- Identify the case number and get the coefficients for the selected pump station.
- Estimate the existing flow rating by using the head differential (ft), engine speed (rpm), and existing rating equation.

Pump station S140_P is used as an example throughout Sections 4 and 5 to demonstrate various steps of the existing flow estimation and evaluation procedures. S140_P is a three-unit pump station, categorized as Case 3. The flow coefficients for pumps at this station are summarized in Table 2.

Table 2. Flow coefficients for pump station S140_P in Case 3 for existing rating equation

| Station | Unit # | $C_{10}$ | $C_{11}$ | $C_{12}$ | $C_{13}$ | $N_{lwr}$ | $N_{upr}$ |
|---|---|---|---|---|---|---|---|
| S140_P | 1 | 409.57 | -19.071 | -4.5714 | 0 | 925 | 1200 |
| | 2 | 409.57 | -19.071 | -4.5714 | 0 | 925 | 1200 |
| | 3 | 409.57 | -19.071 | -4.5714 | 0 | 925 | 1200 |
| Station | Unit # | $C_{20}$ | $C_{21}$ | $C_{22}$ | $C_{23}$ | $C_p$ | |
| S140_P | 1 | 529.52 | -12.454 | -2.6054 | 0 | 0.9 | |
| | 2 | 529.52 | -12.454 | -2.6054 | 0 | 0.9 | |
| | 3 | 529.52 | -12.454 | -2.6054 | 0 | 0.9 | |

The brief descriptions provided here were taken from Atlas of Flow Computations at District Hydraulic Structures (Ansar et al, 2003). The discharge for pump station S140_P in Case 3 is given by (Otero 1995):

$$Q = Q_{lwr} + \left(Q_{upr} - Q_{lwr}\right)\left(\frac{N - N_{lwr}}{N_{upr} - N_{lwr}}\right) \tag{1}$$

where, Q is the discharge at pump speed N; $Q_{lwr}$ and $Q_{upr}$ are the lower and upper discharges at pump speeds $N_{lwr}$ and $N_{upr}$, respectively. $Q_{lwr}$ and $Q_{upr}$ are given by

$$Q_{lwr} = C_{10} + C_{11} \cdot H_{lwr} + C_{12} \cdot H_{lwr}^2 + C_{13} \cdot H_{lwr}^3 \tag{2}$$
$$Q_{upr} = C_{20} + C_{21} \cdot H_{upr} + C_{22} \cdot H_{upr}^2 + C_{23} \cdot H_{upr}^3$$

where, $C_{10}$ through $C_{13}$ and $C_{20}$ through $C_{23}$ are regression coefficients. $H_{lwr}$ and $H_{upr}$ are the heads corresponding to $Q_{lwr}$ and $Q_{upr}$, respectively. $H_{lwr}$ and $H_{upr}$ are obtained from pump affinity laws as follows (Otero 1995):

$$H_{lwr} = H\left(\frac{N_{lwr}}{N}\right)^2 \tag{3}$$
$$H_{upr} = H\left(\frac{N_{upr}}{N}\right)^2$$

where, H is the head differential at pump speed N.

Table 3 shows discharges calculated using the existing flow estimation procedures based on the headwater, tailwater, and engine speed obtained from the streamgauging database (Qmeas) table. The last column in Table 3 indicates the estimated discharges (Q) from the existing rating equations for pump station S140_P.

Table 3. Existing flow estimation for pump station S140_P in Case 3

| Date | HW (ft) | TW (ft) | # of operation | H (ft) | N | Hlwr (ft) | Hupr (ft) | Qlwr (cfs) | Qupr (cfs) | $Q_{computed}$ (cfs) |
|---|---|---|---|---|---|---|---|---|---|---|
| 21-Aug-90 | 9.1 | 11.78 | 2 | 2.68 | 1200 | 1.59 | 2.68 | 367.6 | 477.4 | 954.9 |
| 6-Sep-90 | 8.96 | 11.42 | 1 | 2.46 | 1200 | 1.46 | 2.46 | 371.9 | 483.1 | 483.1 |
| 7-Sep-90 | 9 | 11.31 | 1 | 2.31 | 1100 | 1.63 | 2.75 | 366.2 | 475.6 | 435.8 |
| 12-Sep-90 | 8.91 | 11.35 | 1 | 2.44 | 1100 | 1.73 | 2.90 | 363.1 | 471.4 | 432.0 |
| 28-Sep-90 | 9.22 | 11.19 | 1 | 1.97 | 1200 | 1.17 | 1.97 | 381.0 | 494.9 | 494.9 |
| 1-Oct-90 | 9.02 | 11.29 | 1 | 2.27 | 1100 | 1.61 | 2.70 | 367.2 | 476.9 | 437.0 |
| 21-May-91 | 9.37 | 11.17 | 1 | 1.8 | 1200 | 1.07 | 1.80 | 383.9 | 498.7 | 498.7 |
| 26-Jul-91 | 9.6 | 12.38 | 3 | 2.78 | 1183 | 1.70 | 2.86 | 364.0 | 472.6 | 1398.1 |
| 15-Oct-91 | 9.16 | 11.71 | 1 | 2.55 | 1050 | 1.98 | 3.33 | 353.9 | 459.1 | 401.7 |
| 1-Oct-94 | 8.84 | 13.15 | 2 | 4.31 | 1150 | 2.79 | 4.69 | 320.8 | 413.7 | 793.6 |
| 1-Oct-94 | 8.84 | 13.15 | 3 | 4.31 | 1150 | 2.79 | 4.69 | 320.8 | 413.7 | 1190.4 |
| 1-Oct-94 | 8.85 | 13.13 | 2 | 4.28 | 1150 | 2.77 | 4.66 | 321.7 | 414.9 | 795.9 |
| 4-Oct-94 | 8.91 | 13.16 | 2 | 4.25 | 1100 | 3.01 | 5.06 | 311.0 | 399.9 | 735.1 |
| 12-Oct-94 | 8.91 | 13.16 | 2 | 4.25 | 1100 | 3.01 | 5.06 | 311.0 | 399.9 | 735.1 |
| 2-Nov-94 | 8.68 | 12.9 | 2 | 4.22 | 1050 | 3.28 | 5.51 | 298.1 | 381.7 | 672.2 |
| 6-Nov-98 | 8.95 | 12.77 | 3 | 3.82 | 1150 | 2.47 | 4.16 | 334.5 | 432.6 | 1244.4 |
| 29-Jul-97 | 9.14 | 12.37 | 2 | 3.23 | 1100 | 2.28 | 3.84 | 342.2 | 443.1 | 812.9 |
| 29-Jul-97 | 8.89 | 12.32 | 2 | 3.43 | 1100 | 2.43 | 4.08 | 336.4 | 435.3 | 798.7 |
| 7-Oct-97 | 8.98 | 12.31 | 1 | 3.33 | 1200 | 1.98 | 3.33 | 353.9 | 459.2 | 459.2 |
| 7-Oct-97 | 9.03 | 12.51 | 2 | 3.48 | 1200 | 2.07 | 3.48 | 350.6 | 454.6 | 909.3 |
| 7-Oct-97 | 8.86 | 12.46 | 2 | 3.6 | 1100 | 2.55 | 4.28 | 331.4 | 428.3 | 786.2 |
| 7-Oct-97 | 9.16 | 12.51 | 2 | 3.35 | 1200 | 1.99 | 3.35 | 353.5 | 458.6 | 917.1 |
| 8-Oct-99 | 9.02 | 12.82 | 2 | 3.8 | 1100 | 2.69 | 4.52 | 325.3 | 419.9 | 771.0 |
| 3-Nov-99 | 9.94 | 13.4 | 2 | 3.46 | 1100 | 2.45 | 4.12 | 335.5 | 434.1 | 796.5 |

## 5. Evaluation of Existing Flow Equation

With the increasing number of pump stations in the District, new flow computation models will be needed for the new pump stations. The increase in the number of flow equations will make FLOW more complex and inefficient. It is imperative that we develop a plan to replace these empirical equations by more reliable standard ones that are based on energy principles and pump affinity laws. It is important to evaluate the existing flow equations before developing a new model to replace them.

One way to evaluate the existing rating equation is to compare the measured and computed discharges. The steps of evaluating an existing pump flow equation are:

- Obtain pump performance curves.
- Obtain pump information including date, time, headwater, tailwater, discharge, engine speed, and the number of pumps operating at the time of measurement.
- Evaluate the existing flow equation by assessing the validity of the equation with respect to physical principles, e.g., conservation of energy, continuity equation, and pump affinity laws. By looking at the existing equation, it is possible to see if the equation makes use of these principles. For example, Equation (3) above shows the relationship of engine speed and head differential that is based on the pump affinity laws.
- Run Qverify for the selected pump station. Data verification is performed by comparing the measured flow with the computed flow for each data point, as well as for all data points taken together as a set.
- Running Qmr to get the absolute relative errors for the selected pump station per range of operation.

The evaluation results from Qverify and Qmr are used to determine whether the existing model is suitable or not for the selected pump station. At this stage, it is necessary to decide whether the existing rating is good enough or we should recalibrate it or develop a new one.

## 6. Determination of Need for a New Rating Equation

When enough discharge data points have been obtained, that is, 15 or more points distributed over a wide range of pump operation settings, the measured discharges are compared against the data computed by FLOW. Data verification results are reported in terms of relative errors that help to categorize the correlation of measured data to computed data as excellent, good, fair or poor. The rating is classified as "excellent" when about 95 percent of the predicted flow rates are within 5 percent of the measured discharge, "good" if the flow data are within 10 percent, "fair" if they are within 15 percent and "poor" when they are not within 15 percent (Akpoji et al, 2003). A rating should not be implemented if it is rated less than "fair". A new rating will not be implemented if the improvement of the new rating over the old one is not more than five percent of the measured discharge rates (Akpoji et al, 2003).

The following steps will help to determine if the pump station requires recalibration.

- Examine the results of comparison between measured and computed discharges. If the rating is not classified as good, new calibration equations should be developed. As an example, Table 4 shows the comparison results for pump station S140_P. According to the results shown in Table 4, the average of absolute relative errors (49%) is more than 10% and the percentage of data with absolute relative error within 15% are 69.5% (less than 95%); the existing rating is considered to be not satisfactory and modifications to the existing rating equations or development of a new one will be essential for better flow computation.

- Examine the results of errors per range of operation. If the mean absolute error is more than 10%, new calibration equations must be done. As shown in Table 5, the absolute errors for S140_P are much higher at the higher range of head differentials (between 4.0 and 6.0 ft), and the existing rating equation has to be investigated further and recalibrated.

Table 4. Comparison of measured and computed discharge for S140_P

| Nr | Date | Time | Head Water | Tail Water | Measured Q | Computed Q | Relative Error |
|---|---|---|---|---|---|---|---|
| 1 | 21-Aug-90 | 13:40 | 9.1 | 11.78 | 1348 | 954.861 | -29% |
| 2 | 6-Sep-90 | 13:40 | 8.96 | 11.42 | 471 | 483.116 | 3% |
| 3 | 7-Sep-90 | 13:45 | 9 | 11.31 | 293 | 433.857 | 48% |
| 4 | 12-Sep-90 | 14:08 | 8.91 | 11.35 | 419 | 429.875 | 3% |
| 5 | 28-Sep-90 | 13:35 | 9.22 | 11.19 | 449 | 494.874 | 10% |
| 6 | 1-Oct-90 | 13:55 | 9.02 | 11.29 | 397 | 435.06 | 10% |
| 7 | 21-May-91 | 13:45 | 9.37 | 11.17 | 563 | 498.661 | -11% |
| 8 | 26-Jul-91 | 12:22 | 9.6 | 12.38 | 1255 | 1396.371 | 11% |
| 9 | 15-Oct-91 | 14:38 | 9.16 | 11.71 | 385 | 399.216 | 4% |
| 10 | 1-Oct-94 | 11:00 | 8.85 | 13.13 | 228 | 789.967 | 246% |
| 11 | 1-Oct-94 | 13:15 | 8.84 | 13.15 | 1168 | 1181.431 | 1% |
| 11 | 1-Oct-94 | 13:15 | 8.84 | 13.15 | 1168 | 1181.431 | 1% |
| 12 | 4-Oct-94 | 13:00 | 8.91 | 13.16 | 210 | 725.545 | 246% |
| 13 | 12-Oct-94 | 13:00 | 8.91 | 13.16 | 210 | 725.545 | 246% |
| 14 | 2-Nov-94 | 12:15 | 8.68 | 12.9 | 222 | 661.619 | 198% |
| 15 | 29-Jul-97 | 10:53 | 9.14 | 12.37 | 891 | 806.525 | -9% |
| 16 | 29-Jul-97 | 11:40 | 8.89 | 12.32 | 806 | 791.736 | -2% |
| 17 | 7-Oct-97 | 10:00 | 9.16 | 12.51 | 754 | 917.12 | 22% |
| 18 | 7-Oct-97 | 10:36 | 9.03 | 12.51 | 881 | 909.256 | 3% |
| 19 | 7-Oct-97 | 11:39 | 8.86 | 12.46 | 746 | 778.747 | 4% |
| 20 | 7-Oct-97 | 13:00 | 8.98 | 12.31 | 451 | 459.157 | 2% |
| 21 | 8-Oct-99 | 0:00 | 9.02 | 12.82 | 830.3 | 762.975 | -8% |
| 22 | 3-Nov-99 | 14:13 | 9.94 | 13.4 | 841 | 789.471 | -6% |
| | | | Minimum Relative Error: | | | | -29% |
| | | | Maximum Relative Error: | | | | 246% |
| | | | Average of Relative Error: | | | | 43% |
| | | | Average of Absolute Relative Error: | | | | 49% |
| | | 95% Lower Confidence Interval for the Mean: | | | | | 10.5% |
| | | 95% Upper Confidence Interval for the Mean: | | | | | 75.7% |
| Distribution of Absolute Relative Errors: | | | | | | | |
| Percentage of data with Absolute Relative Error <= 5% is: (Rating is very good) | | | | | | | 39.1% |
| Percentage of data with 5% < Absolute Relative Error <= 10% is: (Rating is good) | | | | | | | 17.4% |
| Percentage of data with 10% < Absolute Relative Error <= 15% is: (Rating is fair) | | | | | | | 13% |
| Percentage of data with Absolute Relative Error > 15% is: (Rating is poor) | | | | | | | 30.5% |
| Number of Records Retrieved from Database: | | | | | | | 23 |
| Number of Records with Valid Flow Estimates: | | | | | | | 23 |

In the case of the high relative error (246%), the measured flow was less than the capacity of one, while two pumps in operation.

Table 5. Absolute error per range of operation for pump station S140_P

| Range of Head Differential, ft (DIFF) | Range of Operation, rpm (RPM) | | | | |
|---|---|---|---|---|---|
| | 800≤RPM≤950 | 950≤RPM≤1100 | 1100≤RPM≤1250 | Abs Error (%) | |
| | | | | Mean | Max |
| 0.0≤DIFF≤2.0 | – | – | 10.82 | 10.82 | 11.43 |
| 2.0<DIFF≤4.0 | – | 10.42 | 11.83 | 11.04 | 48.07 |
| 4.0<DIFF≤6.0 | – | 229.67 | 97.71 | 163.69 | 246.48 |
| Mean | – | 65.24 | 33.13 | 49.18 | – |

## 7. Development of a New Rating Equation

It is necessary to standardize the general rating equations for all pump stations. Standardizing the equations can save time and effort for rating analysis. Pump performance curves are used in conjunction with the principles of energy and mass conservation to develop new rating equations. Figure 2, for example, shows the head-discharge relationship for flows through the pumps at S9_P under laboratory conditions. Various pump speeds are represented by corresponding curves. For the engines in operation after 1989, the top curve represents an engine speed of 733 rpm, the bottom curve represents an engine speed of 660 rpm and the curves in between are for engine speeds between 733 and 660 rpm. For the old engines, i.e. prior to 1989, the top curve represents 400 rpm, the bottom curve represents 360 rpm and the curves in between are for 370, 380 and 390 rpm as shown. The performance curves are parabolic with concave down suggesting that a polynomial function with a power higher than one may be appropriate to compute flow for pumps at S9_P.

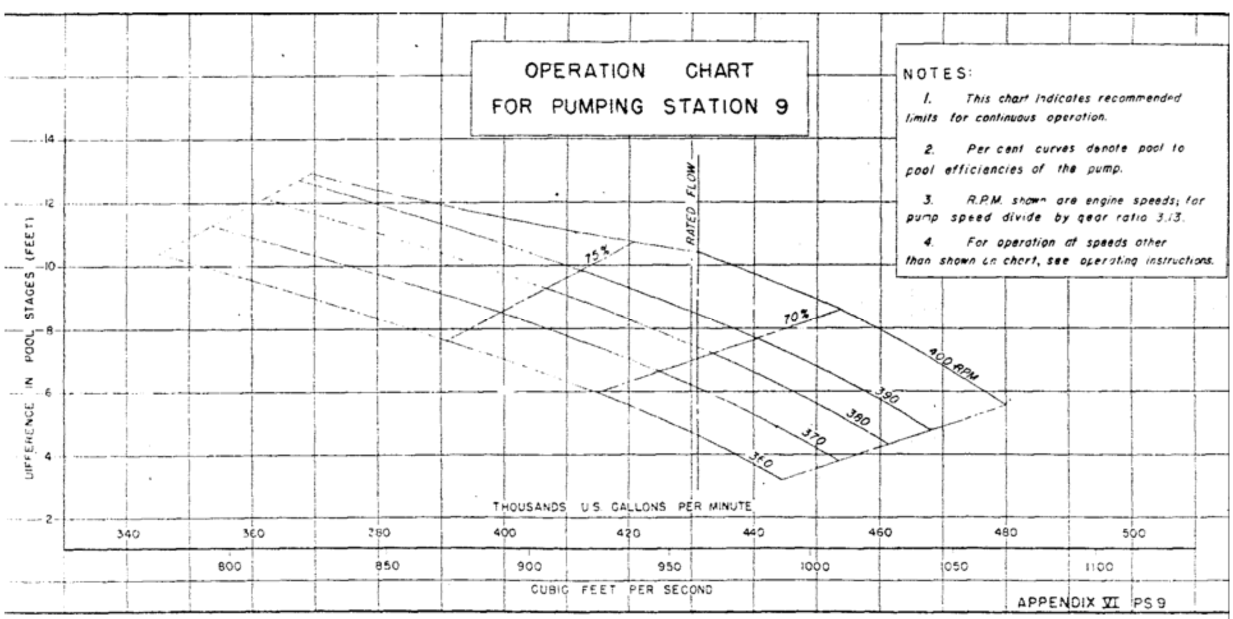


Figure 2. Performance curves for pumps at S9_P

From the energy conservation principle, the velocity is a function of the head differential. Discharge through a constant cross section (such as a pump flow section), which is directly proportional to velocity is a function of the head. The absolute value of the hydraulic head differential (H) is used in all subsequent equations. On the basis of this concept Equation (4) is valid for all Q and H values for the rated pump speed (Imru, 2003).

$$Q_o = f(H) = A + B H_0{}^C \tag{4}$$

In Equation (4), $Q_o$ is the discharge for a reference pump speed, $H_0$ is head differential that corresponds to $Q_o$, A and B are constant coefficients and the value of coefficient B is negative as long as the headwater is lower than the tailwater, and C is a constant exponent and more than one.

The flow rate changes proportionally according to the pump affinity laws when the pump speed varies. The pump affinity laws assume no change in efficiency when engine speed changes and the relation between the change in discharge and the change in pump speed is given by

$$\frac{Q}{Q_0} = \frac{N}{N_0} \tag{5}$$

Substituting Equation (4) into Equation (5) and rearranging, we obtain Equation (6).

$$Q = \frac{N}{N_0}(A + B H_0{}^C) \tag{6}$$

$H_0$ can be written in terms of H using the following relation of the pump affinity laws.

$$H_0 = \left[\frac{N_0}{N}\right]^2 H \tag{7}$$

Substituting Equation (7) in Equation (6) and rearranging, we obtain Equation (8).

$$Q = A\left[\frac{N}{N_0}\right] + BH^C\left[\frac{N_0}{N}\right]^{2C-1} \tag{8}$$

where: Q = Discharge (cfs)
N = Field Measured Engine Speed (rpm)
$N_0$= Design Engine Speed (rpm)
H = Field Measured Head Differential (ft)

Equation (8) presents a model based on physical laws that can be used to estimate flow through variable speed pumps. This equation describes the relationship between discharge, head differential, and engine speed. If the selected pump station needs recalibration based on the previous evaluation results, Equation (8) will be used for the new rating analysis.

## 8. Calibration of the New Rating Equation

Once the calibration field data points have been obtained, a rating analysis is performed to develop a flow equation for the selected pump station. The available measurements and pump performance curves as well as the affinity laws are used to perform the new rating analysis. The discharge at the rated engine speed can be obtained from the field data using the pump affinity laws if needed. The regression coefficients of Equation (4) are determined based on the Least-Squares method (Davis, 1986). According to the Least-Squares method, the deviation of the estimate from the measurement is $((A + B{H_0}^C) - Q_0)$, and our goal becomes one of finding a method such that

$$F = \sum_{i=1}^{n} \left( \left( A + B{H_0}^C \right) - Q_0 \right)^2 = \text{ minimum} \tag{9}$$

The expanded form of above equation is given by

$$F = \sum_{i=1}^{n} \left( {Q_0}^2 - 2AQ_0 - 2B{H_0}^C Q_0 + A^2 + 2AB{H_0}^C + B^2{H_0}^{2C} \right) \tag{10}$$

Mathematically $F$ is minimized by setting its partial derivatives with respect to coefficients A, B, and C equal to zero. The partial derivatives were estimated individually, however, the results show that the three partial derivatives are equivalent and given below

$$\frac{\partial F}{\partial A} = \frac{\partial F}{\partial B} = \frac{\partial F}{\partial C} = \sum_{i=1}^{n} \left( A + B{H_0}^C - Q_0 \right) = 0 \tag{11}$$

$$B = \frac{\sum_{i=1}^{n} Q_0 - nA}{\sum_{i=1}^{n} {H_0}^C} \tag{12}$$

where n is the total number of measurements.

A starting estimation value for coefficient A would be: $A=\sum Q_0/n$. For a parabolic equation, the coefficient A is between the design discharge and the discharge at zero lift. Equation (12) can help to iteratively solve B for the given values of A and C. An iterative simulation helps to determine the optimum values of coefficients A, B, and C for the new rating equation.

To perform rating calibration using both streamflow measurements and pump performance curves, the following steps will help.

- Study the pump performance curves for the selected pump station.
- Analyze the available measurements and investigate the outliers and questionable points, and discard if necessary.
    1. Compare archive data against Qmeas for questionable points.

2. Check if two pumps operate at different speeds at the same time for the same type of pump. Avoid using such a measurement; otherwise, it will cause errors in determining the measured discharge per unit.
3. Check if two different capacity pumps operate at the same time for the same measurements.
4. Check if the available measurements have two different discharge records at the same time, same head water and same tail water.
5. Check if station flow is less than the capacity of one pump, while more than one variable-speed units are operating.
6. Compare the plot of the head differential against discharge with the pump performance curves.

- Plot the head differential against engine speed, preferably data at design pump speed should be used for calibration.
- Use selected measurements (avoiding outliers) for regression analysis to determine the coefficients of the proposed flow equation.

The new pump flow equations must meet the following criteria and all calibration submittals shall provide documentation which indicates these criteria have been met.

- Flow equations should exhibit standard axial flow pump principles (Whalen et al., 1996).
- Application of affinity laws: Head differentials must also be converted when using the affinity laws to convert multiple speed data points to a single speed.
- Predicted flow from discharge equation shall be within ±10% of the measured flows (Whalen et al., 1996).
- $R^2$ should be > 0.85 (Whalen et al., 1996).
- For parabolic equations, coefficient A (zero head differential) must be positive; coefficient B must be negative such that when head differential increases, flow decreases.
- For axial flow pumps, coefficient C should be >1.0 (Damisse, 2000).

In the event that efforts to develop and calibrate a Case 8 equation fails, attempts may be made to use either of the other two equations, the cubic and linear equations as indicated in Appendix D.

Pump station S9_P is used as an example in this Section to show various steps of calibration procedures. All available measurements were used for calibration at pump station S9_P and the rated discharges were obtained using the pump affinity laws at the rated speed (733rpm).

The coefficients and exponents of the new rating equation for S9_P were obtained through regression analysis using available streamgauging data, and Equation (13) presents the new model developed for estimating flow through each diesel pump (Imru, 2003)

$$Q = 1088\left[\frac{N}{N_0}\right] - 2.44H^{1.94}\left[\frac{N_0}{N}\right]^{2.88} \tag{13}$$

Equation (13) is valid when the headwater stage is lower than the tailwater, which is expected to be the most prevalent operating condition.

Table 6 shows the comparison results of measured and computed discharges for the new rating equation at pump station S9_P. The average relative error is 1.3%, with the relative error ranging from -10.2% to 17%, and 95% of all data are within 10% absolute relative errors. Of the forty measurements, the two data points with -10.2% and one with 17% relative errors maybe considered as outliers.

Table 6 Comparison of measured and computed discharges for a new model at S9_P

| MEAS_DATE | TIME | HW | TW | Head | $Qmeas_{ured}$ | N | $Q_{computed}$ | Relative |
|---|---|---|---|---|---|---|---|---|
| | | (ft) | (ft) | (ft) | (cfs) | (rpm) | (cfs) | error |
| 29-May-90 | 12:14 | 1.25 | 7.9 | 6.7 | 1059 | 733 | 992 | -6% |
| 30-May-90 | 12:48 | 0.63 | 7.91 | 7.3 | 1012 | 733 | 973 | -4% |
| 27-Jul-90 | 14:30 | 0.24 | 8.6 | 8.4 | 860 | 733 | 938 | 9% |
| 11-Sep-90 | 14:20 | 0.56 | 9.19 | 8.6 | 886 | 733 | 928 | 5% |
| 14-Sep-90 | 13:00 | 0.44 | 9.02 | 8.6 | 910 | 733 | 930 | 2% |
| 14-Sep-90 | 14:37 | 0.35 | 9.02 | 8.7 | 859 | 733 | 927 | 8% |
| 18-Sep-90 | 14:55 | 0.17 | 8.94 | 8.8 | 884 | 733 | 923 | 4% |
| 27-Sep-90 | 14:10 | 0.34 | 8.82 | 8.5 | 888 | 733 | 934 | 5% |
| 23-May-91 | 12:55 | 1.01 | 8.76 | 7.8 | 897 | 733 | 958 | 7% |
| 23-May-91 | 15:35 | 0.91 | 8.62 | 7.7 | 940 | 733 | 960 | 2% |
| 7-Aug-91 | 13:35 | 0.54 | 10.72 | 10.2 | 742 | 733 | 868 | 17.0% |
| 22-Oct-91 | 17:40 | 3.14 | 10.35 | 7.2 | 897 | 733 | 975 | 9% |
| 22-Jun-94 | 14:40 | 0.98 | 10.1 | 9.1 | 846 | 733 | 910 | 8% |
| 23-Jun-94 | 14:40 | 0.98 | 10.1 | 9.1 | 855 | 733 | 910 | 6% |
| 13-Mar-96 | 12:05 | 1.63 | 10.01 | 8.4 | 728 | 650 | 752 | 3% |
| 13-Mar-96 | 13:44 | 1.63 | 9.85 | 8.2 | 727 | 650 | 759 | 4% |
| 13-Jun-96 | 12:14 | 1.46 | 10.22 | 8.8 | 679 | 650 | 732 | 8% |
| 13-Jun-96 | 13:34 | 1.46 | 10.22 | 8.8 | 719 | 650 | 732 | 2% |
| 15-Jun-97 | 10:40 | 0.96 | 10.84 | 9.9 | 967 | 730 | 873 | -9.7% |
| 15-Jun-97 | 11:40 | 1.11 | 10.84 | 9.7 | 901 | 700 | 809 | -10.2% |
| 15-Jun-97 | 14:29 | 2.52 | 10.58 | 8.1 | 904 | 680 | 836 | -8% |
| 15-Jun-97 | 15:05 | 2.61 | 10.58 | 8.0 | 834 | 650 | 771 | -8% |
| 22-Jun-97 | 9:10 | 0.78 | 10.89 | 10.1 | 667 | 650 | 658 | -1% |
| 22-Jun-97 | 10:22 | 0.95 | 10.83 | 9.9 | 805 | 730 | 873 | 9% |
| 22-Jun-97 | 11:42 | 1.14 | 10.86 | 9.7 | 742 | 690 | 785 | 6% |
| 15-Jan-98 | 13:37 | 0.97 | 10.84 | 9.9 | 836 | 700 | 802 | -4% |
| 1-May-98 | 10:11 | 0.82 | 9.91 | 9.1 | 753.5 | 650 | 715 | -5% |
| 6-Nov-98 | 10:20 | 0.79 | 11.25 | 10.5 | 688.5 | 660 | 666 | -3% |
| 26-Feb-99 | 11:10 | 2.58 | 10.05 | 7.5 | 966 | 700 | 901 | -7% |
| 30-Apr-99 | 11:16 | 1.12 | 9.1 | 8.0 | 958 | 700 | 882 | -8% |
| 30-Apr-99 | 12:07 | 1.1 | 9.06 | 8.0 | 937 | 700 | 883 | -6% |
| 21-May-99 | 14:01 | 1.23 | 9.14 | 7.9 | 758 | 650 | 774 | 2% |
| 2-Jun-99 | 10:38 | 0.94 | 9.44 | 8.5 | 810 | 650 | 746 | -8% |
| 8-Jun-99 | 11:48 | 1.43 | 9.72 | 8.3 | 896 | 700 | 870 | -3% |
| 17-Jun-99 | 12:23 | 0.87 | 9.87 | 9.0 | 711.5 | 650 | 720 | 1% |
| 24-Aug-99 | 12:35 | 1 | 11.12 | 10.1 | 740.5 | 700 | 791 | 7% |
| 24-Aug-99 | 13:15 | 1.02 | 11.14 | 10.1 | 654.5 | 650 | 657 | 0% |
| 26-Aug-99 | 11:13 | 0.73 | 11.09 | 10.4 | 598 | 650 | 643 | 8% |
| 3-Nov-99 | 11:45 | 0.35 | 13.43 | 13.1 | 467.4 | 650 | 459 | -2% |
| 25-Jun-01 | 10:50 | 0.72 | 8.94 | 8.2 | 726.5 | 655 | 771 | 6% |
| | | | | Average Error: | | | | 1% |
| | | | | Minimum Error: | | | | -10.2% |
| | | | | Maximum Error: | | | | 17% |

The two outlier points from the streamgauging tables were checked. Headwater, tailwater, and engine speed data obtained from DCVP (breakpoint flow inputs) were compared against values from streamgauging tables. As shown in Table 7, the flows estimated by using the new model are different for breakpoint input and streamgauging values. The discrepancies may be because of recording errors in stage and engine speed, the effect of the bad weather conditions, or other obstructions in the flow way.

Table 7 Comparison of computed discharges from different input using new rating model

| | | | Data from streamgauging table | | | | | Data from DCVP break point flow input | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Date | TIME | $Q_{measured}$ | HW | TW | N | $Q_{computed}$ | Relative | HW | TW | N | $Q_{computed}$ | Relative |
| | | (cfs) | (ft) | (ft) | (rpm) | (cfs) | error | (ft) | (ft) | (rpm) | (cfs) | error |
| 7-Aug-91 | 13:35 | 742 | 0.54 | 10.7 | 733 | 868 | 17.0% | 0.57 | 10.71 | 733 | 870 | 17.2% |
| 15-Jun-97 | 11:40 | 901 | 1.11 | 10.8 | 700 | 809 | -10.2% | 2.39 | 10.55 | 700 | 875 | -2.8% |

Figure 3 shows head-discharge relationships from measurements and computations (S9_P). The continuous curve represents the manufacturer's curve at 733 rpm, square symbols (red in color) represent field measurements, light crosses (cyan in color) represent computed values using the existing model, dark (dark-blue in color) circles represent flows computed using the new calibrated model. Field measurements as well as calculated values indicate that the actual field performance of the pump is slightly lower than what the manufacturer's curve suggests. This is an expected scenario if the manufacturer's curves are based on model test results under laboratory settings. There may also be reduction in performance due to aging (Imru, 2003).

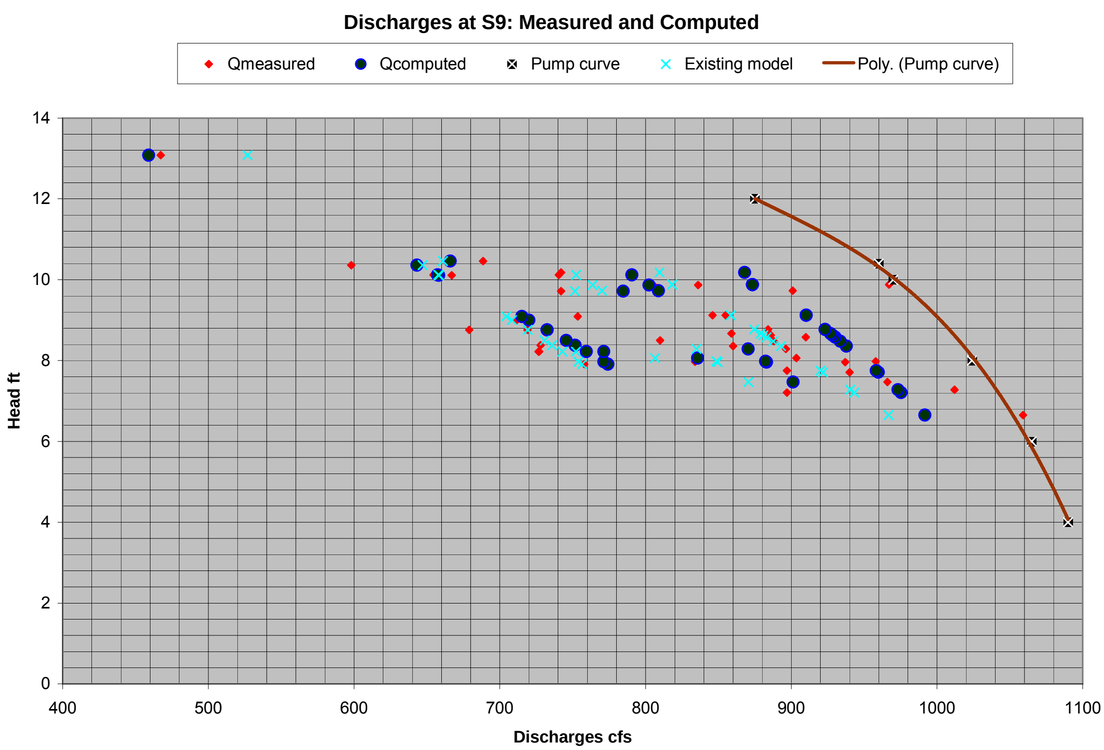


Figure 3 Head and discharge relationship for S9_P resulting from field measurements, existing model, new model ($Q_{computed}$), and 733-rpm theoretical curve (Imru, 2003).

## 9. Conclusions

This document provides the flow rating analysis procedures for pumps through previous sections. The rating analysis report must include available measurements and data analysis, additional measurements required for rating analysis, existing flow estimation procedures, and evaluation of existing flow equations. It should also present a determination on the need for a new rating equation, development of a new rating equation, and calibration of the new rating equation. Finally, the report should provide conclusions, and recommendations of the rating analysis.

Requirement of the additional measurements is based on the available measurements and data analysis results. Determination of need for a new rating equation is according to the results of evaluation of the existing rating equations. For developing a new rating equation, the standard rating equation in case 8 is strongly recommended. A minimum of five, preferable seven, stream flow measurements is required for calibrating the new rating equation. The improvements of the new rating equation, if any, should be addressed in comparison with the existing ones. It should be provided whether flow calculations should continue using the existing rating or be replaced by the new one. The next step in flow monitoring of the pump station under investigation should be stated in recommendation.

## 10. Recommendations

A rating analysis report shall include recommendations on what the next step should do: either keep the existing rating equations, implement the new ones, or do more streamgauging. It is recommended to test and perform impact analyses on all new flow equations before their implementation in Flow.

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# APPENDICES

# APPENDIX A

Homologous (Affinity) Laws

The affinity laws of centrifugal pump performance express the effect on pump performance due to changes in certain application variables. The affinity law variables which affect pump performance are:

1) Pump speed in revolutions per minute (RPM).

2) Impeller diameter.

The impeller diameter of a centrifugal pump is constant. The effect of changing the speed (RPM) of the pump is in accordance with the following:

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2} \tag{1}$$

$$\frac{H_1}{H_2} = \frac{n_1^2}{n_2^2} \tag{2}$$

$$\frac{H_1}{H_2} = \frac{Q_1^2}{Q_2^2} \tag{3}$$

where $Q_1$ is the discharge for a reference pump speed $n_1$, $H_1$ is the head corresponds to $Q_1$.
$Q_2$ is the discharge for a field pump speed $n_2$, $H_2$ is the head corresponds to $Q_2$.

**APPENDIX B**

SQL scripts for all pump stations

```
set pagesize 2500
set linesize 300
column Time format a6 word_wrapped
select distinct x.station, x.meas_date,  to_char(x.meas_date, 'HH:24MI') Time, x.hw_avg HW,
x.tw_avg TW, z.npump Units, x.Discharge Q, y.oper_nr Pump#, r.case_no case, r.pumpdia
pumpdia, y.reading N, r.rpm_noflow Nnoflow,  r.pump_type type, r.unit_no unit
from qm_main x, qm_operations y, dm_pump z, dm_pump_unit r
where x.station=z.station
and   x.station=r.station
and   y.oper_nr=r.unit_no
and y.reading>0
and x.q_meas_id = y.q_meas_id
andx.station like '%_P'
/
```

SQL scripts for pump station S140_P

```
set pagesize 2500
set linesize 300
column Time format a6 word_wrapped
select  distinct x.station, x.meas_date,  to_char(x.meas_date, 'HH24:MI') Time, x.hw_avg HW,
x.tw_avg TW, z.npump Units, x.Discharge Q, y.oper_nr Pump#, r.case_no case, r.pumpdia
pumpdia, y.reading N, r.rpm_noflow Nnoflow,  r.pump_type type, r.unit_no unit
from qm_main x, qm_operations y, dm_pump z, dm_pump_unit r
where x.station=z.station
and x.station=r.station
and y.oper_nr=r.unit_no
and y.reading>0
and x.q_meas_id = y.q_meas_id
andx.station = 'S140_P'
/
```

## APPENDIX C

Existing Flow Equations for Pump Stations

For flow computation purposes the District pumps and corresponding flow equations are classified into eight cases. For Cases1 through 7, the brief descriptions and equations provided here were taken from Atlas of Flow Computations at District Hydraulic Structures (Ansar et al, 2003) and those for Case 8 were taken from Rating Analysis for Pump Station S9 (Imru, 2003).

Case 1. Constant-Speed Pumps

$$Q = C_0 + C_1 \cdot H + C_2 \cdot H^2 + C_3 \cdot H^3 \tag{1}$$

where, Q is the discharge rate in cfs, $C_0$ through $C_3$ are regression coefficients, and H is the head difference between upstream and downstream stage in ft. Pumps in Case 1 are listed in Table 1.

Table 1. Pumps in Case1 for pump stations

| | | | | | | |
|---|---|---|---|---|---|---|
| Station | ACME2 | G123_P | G200A_P | G200B_P | G201_P | G207 |
| Unit # | 1-3 | 1-4 | 1-3 | 1-3 | 1-3 | 1 |
| Station | G208 | G210_P | G349_P | G349B_P | G350A_P | G350B_P |
| Unit # | 1 | 1 | 1-2 | 1 | 1-2 | 1 |

Case 2. Highly-Variable Speed Pumps

The discharge in this case is given by a third-order model with two independent variables.

$$\begin{aligned} Q = {} & C_0 + C_1 \cdot X + C_2 \cdot Y + C_3 \cdot X^2 + C_4 \cdot XY + C_5 \cdot Y^2 + C_6 \cdot X^3 + \\ & C_7 \cdot YX^2 + C_8 \cdot XY^2 + C_9 \cdot Y^3 \end{aligned} \tag{2}$$

where, Q is the discharge in cfs; $C_0$ through $C_9$ are regression coefficients; X is the ratio of the head H in feet and the head factor Hfact in feet, i.e., X = H/Hfact; Y is a dimensionless engine speed parameter given by Y = (N-Nmin)/Nfact; where, N is the engine speed in rpm; Nfact is the engine speed factor, Nfact = Nmax-Nmin; Nmin and Nmax are, respectively, the minimum and maximum engine speed. Pumps in Case 2 are summarized in Table 2.

Table 2. Pumps in Case 2 for pump stations

| Station | S127_P | S129 PMP_P | S131 PMP_P | S2_P | S3 PMP_P | S3_P |
|---|---|---|---|---|---|---|
| Unit # | 1-5 | 1-3 | 1-2 | 1-4 | 1-3 | 1-3 |

Case 3. Variable-Speed Pumps

The discharge in this case is given by:

$$Q = Q_{lwr} + \left(Q_{upr} - Q_{lwr}\right)\left(\frac{N - N_{lwr}}{N_{upr} - N_{lwr}}\right) \tag{3}$$

where, Q is the discharge at pump speed N; $Q_{lwr}$ and $Q_{upr}$ are the lower and upper discharges at pump speeds $N_{lwr}$ and $N_{upr}$, respectively. $Q_{lwr}$ and $Q_{upr}$ are given by

$$\begin{aligned} Q_{lwr} &= C_{10} + C_{11} \cdot H_{lwr} + C_{12} \cdot H_{lwr}^{2} + C_{13} \cdot H_{lwr}^{3} \\ Q_{upr} &= C_{20} + C_{21} \cdot H_{upr} + C_{22} \cdot H_{upr}^{2} + C_{23} \cdot H_{upr}^{3} \end{aligned} \tag{4}$$

where, $C_{10}$ through $C_{13}$ and $C_{20}$ through $C_{23}$ are regression coefficients. $H_{lwr}$ and $H_{upr}$ are the heads corresponding to $Q_{lwr}$ and $Q_{upr}$, respectively. $H_{lwr}$ and $H_{upr}$ are obtained from pump affinity laws as follows (Otero 1995):

$$\begin{aligned} H_{lwr} &= H\left(\frac{N_{lwr}}{N}\right)^{2} \\ H_{upr} &= H\left(\frac{N_{upr}}{N}\right)^{2} \end{aligned} \tag{5}$$

H is the head at pump speed N. Pumps in Case 3 are summarized in Table 3.

Table 3. Pumps in Case 3 for pump stations

| | | | | | |
|---|---|---|---|---|---|
| Station | S133_P | S135 PMP_P | S236_P | S331_P | S4_P |
| Unit # | 1-5 | 1-4 | 1-3 | 1-3 | 1-3 |
| Station | S5A_P | S6_P | S7_P | S8_P | S140_P |
| Unit # | 1-6 | 1-3 | 1-3 | 1-4 | 1-3 |

Case 4. Highly-variable speed pump with two versions of flow algorithms

Case 4 makes use of the flow equations for Case 2, i.e., a two-variable polynomial is used to model the flow. However, the flow is computed using two versions, each with its own set of flow coefficients. This case is used at pump station S9 which is a three unit pump station. Pumps in Case 4 are summarized in Table 4.

Table 4. Pumps in Case 4 for pump station

| | |
|---|---|
| Station | S9_P |
| Unit # | 1-3 |

Case 5. Constant-speed pump with the possibility of an unsubmerged outlet

For Case 5 a second-order polynomial is used to compute discharge, i.e.,

$$Q = C_0 + C_1 \cdot H + C_2 \cdot H^2 \tag{6}$$

where, Q is the discharge in cfs; $C_0$ through $C_2$ are regression coefficients, and H is the head difference between upstream and downstream stages in ft. Pumps in Case 5 are listed in Table 5.

Table 5. Pumps in Case 5 for pump stations

| Station | G250S_P | G250_P | G251_P | S332_P |
|---|---|---|---|---|
| Unit # | 1-3 | 1-6 | 1-6 | 1-9 |

Case 6. Variable-speed Pumps at G600 and ACME1

In this case the flow is computed as a function of pump speed and head, and is given by:

$$Q = 0.00223 \cdot C_2 \cdot \frac{N}{C_0} \cdot \left\{ C_1 - \left[ \frac{\left(\frac{C_0}{N}\right)^2 \cdot H - C_3}{\left| \left(\frac{C_0}{N}\right)^2 \cdot H - C_3 \right|} \right] \left[ \left| \left(\frac{C_0}{N}\right)^2 \cdot H - C_3 \right| \right]^{1/3} \right\} \tag{7}$$

where, Q is the discharge in cfs; $C_0$ through $C_3$ are regression coefficients; H is the head difference between upstream and downstream stages in ft; and N is the pump speed in rpm. Pumps in Case 6 are listed in Table 6.

Table 6. Pumps in Case 6 for pump stations

| Station | G600_P | ACME1 |
|---|---|---|
| Unit # | 1-5 | 1-3 |

Case 7. Pumps at S13 and S332D

The flow equations in this case were developed from pump affinity laws (Imru 1999) and are given by:

$$Q = \left(\frac{N}{N_R}\right)\left[C_1\sqrt{H} + C_3\right] \text{ if HW} < \text{TW} \tag{8}$$

and

$$Q = \left(\frac{N}{N_R}\right)\left[C_2\sqrt{H} + C_4\right] \text{ if HW} \geq \text{TW}$$

where, Q is the discharge in cfs; $N_R$ is the rated rpm of the pump; N is the pump speed; $C_1$ through $C_4$ are regression coefficients; HW and TW are, respectively the headwater and tailwater elevations in ft; and H is the absolute head differential, i.e., H = |HW-TW|. Pumps in Case 7 are listed in Table 7.

Table 7. Pumps in Case 7 for pump stations

| Station | S13_P | S332D_P |
|---|---|---|
| Unit # | 1-3 | 1-5 |

Case 8. Pumps at S310 and S335

The flow equations in this case were developed from pump affinity laws and are given by:

$$Q = A\left[\frac{N}{N_0}\right] + BH^C\left[\frac{N_0}{N}\right]^D \tag{9}$$

where, Q is the discharge in cfs; $N_0$ is the rated rpm of the pump; N is the pump speed; A, B, C, and D are regression coefficients; and H is the absolute head differential. Pumps in Case 8 are list in Table 8.

Table 8. Pumps in Case 8 for pump stations

| Station | G310_P | G335_P | G404_P | G409_P | G410_P | S332B_P |
|---|---|---|---|---|---|---|
| Unit # | 1-6 | 1-6 | 1-3 | 1-3 | 1-2 | 1-5 |

## APPENDIX D

Graphical Representations for Various Types of Flow Equations (Cases)